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THE DETERMINATION OF $|V_{cb}|$ AND QCD SUM RULES IN HQET⋆

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Abstract

I review recent developments in Heavy Quark Effective Theory (HQET) that lead to an almost model–independent determination of the $|V_{cb}|$ element of the Cabibbo–Kobayashi–Maskawa matrix from exclusive semileptonic $B \to D^{(*)}$ decays. In particular, I compare the theoretical uncertainties in the $B \to D^* \ell\, \bar\nu$ and the $B \to D\, \ell\, \bar\nu$ decay modes. I discuss the applications of QCD sum rules within HQET to semileptonic heavy meson decays and give predictions for the form factors measurable in $B \to D^{(*)} \ell\, \bar\nu$ decays.

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1. Introduction

In the absence of direct observations of new physics, heavy hadron decays may provide the first clues to physics beyond the Standard Model (SM). They probe the flavor sector, which contains the majority of the parameters of the SM. These parameters have to satisfy certain relations provided by, *e.g.*, the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, and the SM prescription of CP violation. Measuring the parameters of the SM more accurately, one hopes to find inconsistencies among them, which would give hints to new physics. Even if the numerical values of these parameters are consistent with each other, they may provide insights to new physics that can yield relations among them. (For example, schemes for quark mass matrices can be tested, that may teach us about horizontal symmetries or grand unification.) Certain rare decays are particularly sensitive probes of various extensions of the SM. However, the theoretical predictions for most of these measurements are contaminated by large hadronic uncertainties. Reducing these uncertainties would provide better chances of discovering physics beyond the SM, and make the bounds on such new physics more restrictive. Recent developments in this direction constitute the subject of this talk.

In hadrons composed of one heavy quark and a number of light degrees of freedom (gluons and light quarks), the energy scale of strong interactions is small compared to the heavy quark mass. The heavy quark acts effectively as a static source of color, resulting in new symmetries of QCD [1–7]: the interaction between the heavy quark and the surrounding light degrees of freedom become independent of the mass and spin of the heavy quark. In the $m_Q \gg \Lambda_{\rm QCD}$ limit the velocity of the heavy quark is conserved with respect to soft processes [1,7], and the complexity of hadronic dynamics results from the strong interactions among the light degrees of freedom only. From the phenomenological point of view, such a symmetry (even if broken as the 'heavy' quarks are, after all, not infinitely heavy) is an extremely useful tool, as it provides exact predictions in the symmetry limit, reducing hadronic uncertainties and model–dependence, which can only enter in corrections, suppressed by powers of $1/m_Q$.

The heavy quark effective theory (HQET) [5–11,13] provides a convenient framework to analyze heavy hadron decays. It allows for a systematic expansion of hadronic quantities in powers of $\Lambda_{\rm QCD}/m_Q$ in such a way that the coefficients are heavy quark spin– and mass–independent universal functions of the kinematic variable $y = v\cdot v'$, where v and v' denote the velocities of the initial and final hadrons. These universal functions originate from long distance QCD, so they can only be investigated using nonperturbative methods. Such a method is provided by QCD sum rules [14–16], which have been widely used recently to calculate hadronic matrix elements in HQET [17–24].

In this talk I shall focus on decays of heavy mesons (rather than baryons). Experimentally they are easier to measure, resulting in more phenomenological applications (*e.g.*, extraction of $|V_{cb}|$), and most calculations of HQET form factors using either QCD sum rules or other

models have been carried out for these decays.[†] Moreover, in heavy meson decays a comparison with lattice calculations is also possible [27].

In Section 2, I review aspects of HQET that are relevant to the discussion in Section 3 of the almost model–independent determination of $|V_{cb}|$ from exclusive semileptonic B meson decays. In Section 4, I discuss applications of QCD sum rules for heavy meson decays. Besides the universal functions, predictions for heavy meson decay constants and semileptonic form factors are also presented. Finally, I summarize and outline some directions of ongoing (and future) developments.

2. HQET

The construction of HQET starts with removing the mass–dependent piece of the momentum operator by introducing a field $h_Q(v,x)$, which annihilates a heavy quark with velocity v [7],

$$h_Q(v,x) = e^{im_Q v\cdot x}\, P_+(v)\, Q(x), \tag{1}$$

where $P_+(v) = \frac{1}{2}(1+\not v)$ is an on–shell projection operator onto the heavy quark (rather than antiquark) components of the spinor, and $Q(x)$ denotes the conventional quark field in QCD. If P^μ is the total momentum of the heavy quark, the new field h_Q carries the residual momentum $k^\mu = P^\mu - m_Q\, v^\mu \sim \vee(\Lambda_{\rm QCD})$, which does not grow with the heavy quark mass.

In the limit $m_Q \gg \Lambda_{\rm QCD}$, the effective Lagrangian for the strong interactions of the heavy quark is [5–8]

$$\mathcal{L}_{\rm HQET} = \bar h_v\, iv\cdot D\, h_v + \frac{1}{2m_Q}\Big[O_{\rm kin} + C_{\rm mag}(\mu)\, O_{\rm mag}\Big] + \vee(1/m_Q^2)\,, \tag{2}$$

where $D^\mu = \partial^\mu - ig_s T_a A_a^\mu$ is the gauge–covariant derivative. The leading term respects both the spin and flavor symmetries. The operators appearing at order $1/m_Q$ are

$$O_{\rm kin} = \bar h_v\,(iD)^2 h_v\,, \qquad O_{\rm mag} = \frac{g_s}{2}\,\bar h_v\,\sigma_{\mu\nu} G^{\mu\nu} h_v\,. \tag{3}$$

Here $G^{\mu\nu}$ is the gluon field strength tensor defined by $[iD^\mu, iD^\nu] = ig_s G^{\mu\nu}$. In the hadron's rest frame, $O_{\rm kin}$ describes the kinetic energy resulting from the residual motion of the heavy quark, whereas $O_{\rm mag}$ corresponds to the chromomagnetic coupling of the heavy quark spin to the gluon field. While $O_{\rm kin}$ violates only the heavy quark flavor symmetry, $O_{\rm mag}$ violates the spin symmetry as well. Due to reparameterization invariance [28] $O_{\rm kin}$ is not renormalized to all orders in perturbation theory, while $C_{\rm mag}(\mu)$ is a renormalization factor for $O_{\rm mag}$. The heavy quark symmetries are also manifest in the Feynman rules of the effective theory: the propagator of a heavy quark becomes independent of its mass (flavor symmetry) and no gamma matrix appears in the coupling of a heavy quark to the gluon field (spin symmetry).

† The interested reader can find the HQET formalism for heavy baryon decays in Ref. [25]; the QCD sum rules determination of the relevant universal function in [26].

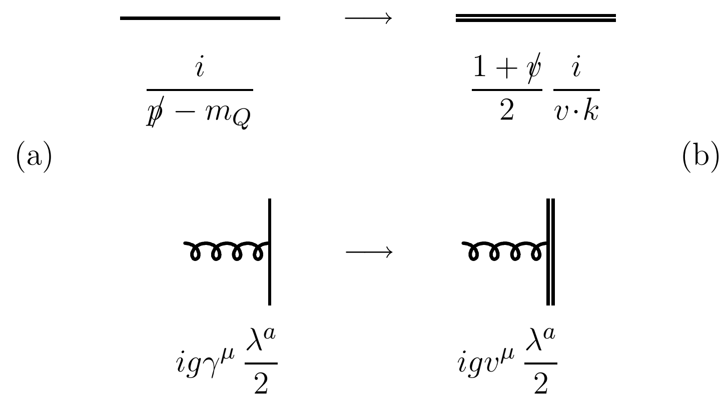

Figure 1: Feynman rules in "full" QCD (a) and in HQET (b). Double lines denote heavy quark propagators in HQET (from [29]).

Any operator of the full theory that contains one or more heavy quark fields can be matched onto a short distance expansion in terms of operators of the effective theory. In particular, the expansion of the heavy quark current $\bar{Q}'\,\Gamma\,Q$ that gives rise to $B \to D^{(*)}$ decays is matched onto

$$\bar{Q}'\,\Gamma\,Q \to \sum_i C_i(\mu)\,J_i + \sum_j \left[\frac{B_j(\mu)}{2m_Q} + \frac{B'_j(\mu)}{2m_{Q'}}\right] O_j + \vee(1/m_Q^2)\,. \tag{4}$$

The operators $\{J_i\}$ form a complete set of local dimension three current operators with the same quantum numbers as the current in the full theory. There are three such operators $J_i = \bar{h}_{Q'}\,\Gamma_i\,h_Q$, with $\Gamma_i = \{\gamma^\mu,\, v^\mu,\, v'^\mu\}$ for the vector current, and $\Gamma_i = \{\gamma^\mu\gamma_5,\, v^\mu\gamma_5,\, v'^\mu\gamma_5\}$ for the axial current. (In the leading logarithmic approximation only $\Gamma = \gamma^\mu\,(\gamma_5)$ contributes. Radiative corrections induce the other operators.) Similarly, $\{O_j\}$ denote a complete set of local dimension four operators. Since there are fourteen independent such operators, we do not display them explicitly here. These effective current operators have non–zero anomalous dimensions. The coefficients $C_i(\mu)$ and $B_j^{(\prime)}(\mu)$ ensure that the final result for any physical quantity is independent of the renormalization procedure. At present, the expansion of the effective Lagrangian and the weak currents are known in perturbation theory up to order α_s/m_Q and $1/m_Q^2$ [13].

2.1. Leading order

Matrix elements in HQET are conveniently calculated in the compact trace formalism [9,10], where a heavy meson is represented by its spin wave function

$$\mathcal{M}(v) = \sqrt{m_Q}\,\frac{(1+\not v)}{2}\begin{cases} -\gamma_5 & ;\ J^P = 0^-, \\ \not\epsilon & ;\ J^P = 1^-, \end{cases} \tag{5}$$

which has the correct transformation properties under Lorentz boosts and heavy quark spin rotations. When the external weak current changes $v \to v'$ (and maybe $Q \to Q'$), the light

degrees of freedom have to rearrange themselves, which yields a form factor suppression. Due to heavy quark symmetry, this suppression factor cannot depend on the spin and the mass of the heavy quark, neither on the Dirac structure of the current. Lorentz and parity invariance, and the properties of $\mathcal{M}(v)$ imply that dependence only on $y = v\cdot v'$ and on the renormalization scale μ is allowed. Hence, a single universal, *i.e.* only y and μ dependent, function $\xi(y,\mu)$ is sufficient to parameterize all semileptonic $M(v) \to M'(v')\,\ell\,\bar{\nu}$ decays, where M and M' are pseudoscalar or vector mesons containing a single heavy quark:

$$\left\langle M'(v')\right|\bar{h}'_{v'}\,\Gamma\,h_v\left|M(v)\right\rangle = -\xi(y,\mu)\ \mathrm{Tr}\left\{\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\right\}. \tag{6}$$

Vector current conservation implies that when the heavy meson in the final state is at rest in the rest frame of the decaying heavy meson, this so–called Isgur–Wise function satisfies $\xi(1) = 1$. The predictions of HQS are most restrictive at this special kinematic point ("zero recoil": $y = 1$), allowing model–independent predictions, unaffected by hadronic uncertainties.

Thus, at leading order in the heavy quark expansion, matrix elements factorize into a kinematic part that depends on the mass and the spin–parity of the mesons, and a reduced matrix element that describes the light degrees of freedom. This is a remarkable simplification, as *a–priori* six independent form factors describe the semileptonic $B \to D^{(*)}$ transitions. Since the b and c quarks are not much heavier than Λ_{QCD}, an analysis of the $1/m_Q$ corrections is important for most phenomenological applications.

2.2. $1/m_Q$ CORRECTIONS

At subleading order, matrix elements receive contributions from the higher dimension operators in the effective Lagrangian (2) and in the effective current (4). The idea is to leave the heavy quark propagator identical to its leading order expression and account for the correction terms in the Lagrangian as insertions of operators. To parameterize their matrix elements we need three new universal functions $\chi_i(y)$ ($i = 1,2,3$). Vector current conservation implies $\chi_1(1) = \chi_3(1) = 0$ (this is known as Luke's theorem [11]).

Matrix elements of the $1/m_Q$ corrections in the effective current (4) are parameterized in terms of another three universal form factors, usually denoted by $\xi_+(y)$, $\xi_-(y)$, and $\xi_3(y)$. Imposing the equation of motion, $i(v\cdot D)h_Q = 0$, on the matrix element yields two constraints [11]. Thus only one of these three functions, say $\xi_3(y)$, is independent.

function	$\xi(y)$	$\chi_1(y)$	$\chi_2(y)$	$\chi_3(y)$	$\xi_3(y)$
normalization	$\xi(1) = 1$	$\chi_1(1) = 0$	no	$\chi_3(1) = 0$	no
broken symmetries	no	flavor	spin, flavor	spin, flavor	spin, flavor

Table 1: Properties of the universal functions of HQET.

So already at order $1/m_Q$ one encounters a set of four universal functions $\xi_3(y)$ and $\chi_i(y)$ $(i = 1, 2, 3)$ in addition to the Isgur–Wise function, as well as a parameter $\bar{\Lambda} = m_M - m_Q$ that describes the mass difference between a heavy meson and the heavy quark that it contains [11,30]. This parameter sets the scale of the $1/m_Q$ expansion; in fact, the real expansion parameter is $\bar{\Lambda}/2m_Q$. The universal form factors are real due to T invariance of the strong interaction. Knowledge of these functions would teach us about confinement and enhance the phenomenological applications of the heavy quark expansion.

3. Model Independent Determination of $|V_{cb}|$

The magnitude of the CKM matrix element V_{cb} can best be determined from an extrapolation of the semileptonic B decay rate to zero recoil, making use of the known normalization of the Isgur–Wise function at that point [31]. The advantage of this method over previous determinations of $|V_{cb}|$ is that in the framework of HQET a clear separation between the model–independent and model–dependent ingredients of the analysis is possible, due to a systematic expansion in the small parameters $\bar{\Lambda}/2m_{c,b}$.

The $B \to D^{(*)}\ell\,\bar{\nu}$ differential decay rate near zero recoil is given by,

$$\begin{aligned}
\lim_{y\to 1} \frac{d\Gamma(B \to D^*\ell\,\bar{\nu})}{dy} &= \frac{G_F^2}{4\pi^3}\, m_{D^*}^3 (m_B - m_{D^*})^2\, |V_{cb}|^2\, (y^2-1)^{1/2}\, \eta^{*2}, \\
\lim_{y\to 1} \frac{d\Gamma(B \to D\,\ell\,\bar{\nu})}{dy} &= \frac{G_F^2}{48\pi^3}\, m_D^3 (m_B + m_D)^2\, |V_{cb}|^2\, (y^2-1)^{3/2}\, \eta^2.
\end{aligned} \tag{7}$$

On the right hand sides of these relations the Fermi constant and the meson masses are well known quantities, powers of (y^2-1) arise from phase space, $\eta^{(*)}$ is defined to include all hadronic uncertainties at $y = 1$, and we want to extract $|V_{cb}|$. The kinematic variable y is related to the conventional q^2 via

$$y \equiv v \cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2\, m_B\, m_{D^{(*)}}}\,. \tag{8}$$

In this notation maximal q^2 corresponds to $y = 1$, while $q^2 = 0$ corresponds to maximal y, which is about 1.5 and 1.6 in $B \to D^*$ and $B \to D$ decays respectively. The problem is that *a–priori* we know nothing about $\eta^{(*)}$, except that it should be of order one. The power of HQS is that it gives the model–independent prediction in the infinite quark mass limit: $\lim_{m_Q\to\infty} \eta^{(*)} = \xi(1) = 1$. This allows us to write

$$\eta^{(*)} = 1 + \delta_{\alpha_s}^{(*)} + \delta_{1/m_Q}^{(*)} + \delta_{1/m_Q^2}^{(*)} + \text{higher order}\,. \tag{9}$$

We shall discuss each of the correction terms in the sequel. The calculation of the perturbative QCD corrections must include the full order α_s terms (not just the leading logarithms), because

$\ln(m_b/m_c) \sim 1.2$ is not a big number. We emphasize that these corrections do not introduce uncertainty into the analysis. They are given by $\delta_{\alpha_s} = 0.05$ and $\delta^*_{\alpha_s} = -0.01$ [32]. While the $B \to D^*\ell\,\bar\nu$ decay rate is protected against $1/m_Q$ corrections at zero recoil due to Luke's theorem [11], *i.e.* $\delta^*_{1/m_Q} = 0$, the $B \to D\,\ell\,\bar\nu$ decay is not, due to its helicity suppression [12,31]. In this latter case the $1/m_Q$ corrections are given by

$$\delta_{1/m_Q} = \left(\frac{\bar\Lambda}{2m_c} + \frac{\bar\Lambda}{2m_b}\right)\left(\frac{m_B - m_D}{m_B + m_D}\right)^2 [1 - 2\xi_3(1)]\,. \tag{10}$$

Clearly, the form factor $\xi_3(y)$ is very important for the determination of $|V_{cb}|$ from $B \to D\,\ell\,\bar\nu$ decays. For example, if $\xi_3(1)$ were around -1 then this $1/m_Q$ correction would be about 15%, while if it were around 0.5 then the $1/m_Q$ correction would vanish. In the next section we shall see that QCD sum rules predict $\xi_3(1) = 0.6 \pm 0.2$ [24], which implies that the $1/m_Q$ correction to the $B \to D\,\ell\,\bar\nu$ decay rate at zero recoil is not more than 3%. This is a result of two suppression factors (beyond $\bar\Lambda/2m_Q$): the Voloshin–Shifman factor $[(m_B - m_D)/(m_B + m_D)]^2 \simeq 0.23$ [4], and an accidental suppression factor of $[1 - 2\xi_3(1)] \sim 0.2$. The $1/m_Q^2$ corrections are expected to be 3–4% on dimensional grounds, and the detailed analysis of Ref. [33] supports that these corrections are not larger than the above estimate. (This statement, however, is somewhat model–dependent, which leaves room for arguments that these corrections might be larger.) The higher order corrections are certainly negligible, *e.g.* the characteristic size of the second order QCD corrections is $[\alpha_s(m_c)/\pi]^2 < 1\%$. Thus the $1/m_Q$ correction to the $B \to D\,\ell\,\bar\nu$ decay rate at zero recoil is not more than the expected $1/m_Q^2$ corrections. This suggests that the theoretical uncertainty in the determination of $|V_{cb}|$ from $B \to D$ transition is comparable to that in $B \to D^*$, even though the latter appears only at order $1/m_Q^2$. Of course, the experimental measurement of $B \to D\,\ell\,\bar\nu$ near zero recoil is more difficult due to extra power of $(y-1)$ helicity suppression in Eq. (7). The reward of such a measurement, however, would be an independent determination of $|V_{cb}|$ with surprisingly small theoretical uncertainties.

A different kind of uncertainty enters into the analysis because phase space vanishes at $y = 1$. Therefore, an extrapolation of the measured spectrum to zero recoil is needed to obtain the numerical value of $|V_{cb}|$. At present this gives rise to both a theoretical and an experimental error. The former is due to the fact that the precise shape of the Isgur–Wise function is not known, while the latter is dominated by statistical error. However, in the not–so–distant future this theoretical uncertainty will almost disappear, since in an asymmetric B factory the zero recoil limit does not correspond to the D^* meson being at rest in the laboratory frame. Then the pion in the subsequent $D^* \to D\,\pi$ decay is boosted, while it is almost at rest for ARGUS and CLEO. In addition, the $(y^2-1)^{1/2}$ phase space suppression is a very mild one: the statistical error of measuring the rate at $y = 1.05$ is less than a factor of two higher than that at the endpoint $y_{\rm max} \simeq 1.5$. At present any information is important on the shape of the Isgur–Wise function, in particular on its slope at $y = 1$, to make the extraction of $|V_{cb}|$ more reliable. Global quark–hadron duality (that the sum of probabilities to decay into hadrons equals to the

probability of free quark transition when $m_Q \to \infty$) gives the Bjorken [10] and Voloshin [34] sum rules:

$$\frac{1}{4} < \rho^2 \lesssim 1\,, \tag{11}$$

where ρ^2 is minus the slope of the Isgur–Wise function at $y = 1$. For a discussion of the bounds derived by de Rafael and Taron [35] we refer to [36].

In summary, we emphasize that due to heavy quark symmetry the theoretical prediction for the $B \to D^{(*)}\ell\,\bar\nu$ decay rate near zero recoil is very precise:

$$\begin{aligned} \eta^* &= 1 + \delta^*_{\alpha_s} + \delta^*_{1/m_Q^2} = 0.99 \pm 0.05\,, \\ \eta &= 1 + \delta_{\alpha_s} + \delta_{1/m_Q} + \delta_{1/m_Q^2} = 1.05 \pm 0.08\,. \end{aligned} \tag{12}$$

Experimentally only the $B \to D^*\ell\,\bar\nu$ spectrum has been measured (for $B \to D\,\ell\,\bar\nu$ only the total decay rate is known). The following figure shows the most recent data from the CLEO collaboration [37].

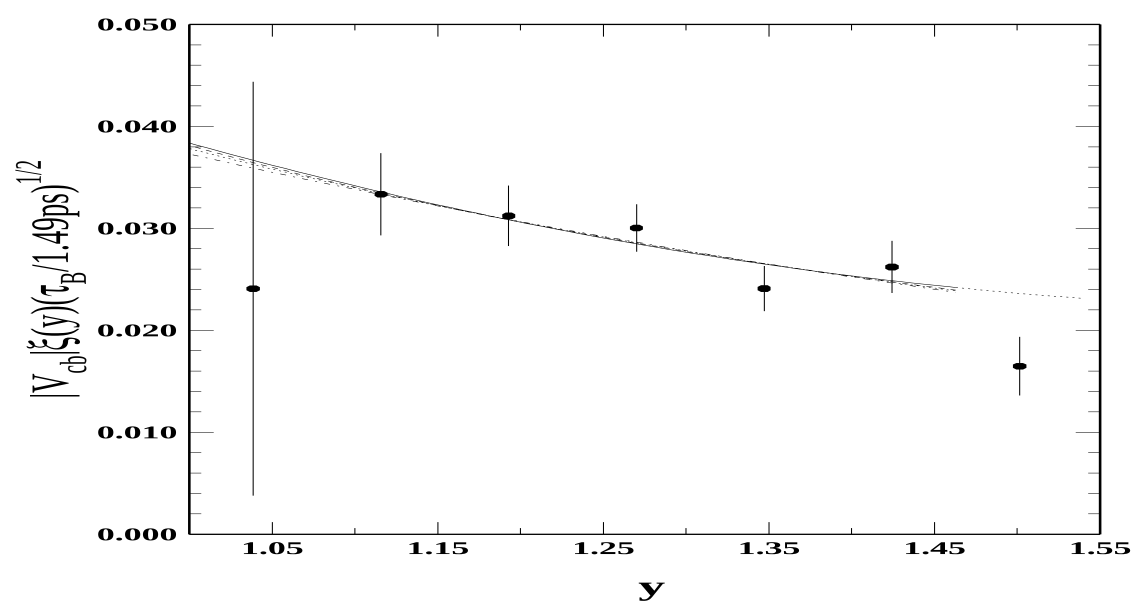

Figure 2: The measured $d\Gamma(B \to D^*\ell\,\bar\nu)/dy$ distribution. The curves represent fits with different functional forms for the Isgur–Wise function.

They obtain from a fit to this distribution

$$\begin{aligned} |V_{cb}| &= 0.037 \pm 0.005 \pm 0.004\,, \\ \rho^2 &= 1.0 \pm 0.4 \pm 0.2\,. \end{aligned} \tag{13}$$

As the theoretical uncertainty in the determination of $|V_{cb}|$ is significantly smaller than the roughly 15% systematic plus statistical error at present – we should look forward to new data from CLEO and a future B factory.

4. QCD Sum Rules

We have seen, so far, two important examples when more information is needed about the universal functions describing heavy meson decays than what is provided by vector current conservation (see: Table 1). The shape of the Isgur–Wise function is important for the extraction of $|V_{cb}|$, and knowledge of $\xi_3(1)$ is essential for the determination of $|V_{cb}|$ from the decay $B \to D\,\ell\,\bar{\nu}$. These universal functions originate from the long distance confining interactions, so they can only be investigated using nonperturbative approaches to QCD. While short distance QCD is well understood, there is still no accurate quantitative framework for dealing with the long distance, strong interaction regime. Lattice gauge theory is the only method known that comes from first principles and has the hope of achieving arbitrary accuracy. This is hindered, however, at present, by the technical limitations of this approach. A less fundamental method is provided by QCD sum rules [14–16], which is particularly suited for the calculation of HQET form factors. It has the advantage over other models that it does not rely on *ad hoc* assumptions, the errors can be estimated, and there is a consistency check on its assumptions within the method.

In the rest of this section we first describe the QCD sum rule method through the example of calculating the heavy meson decay constant in HQET, then give the results for the universal functions, and finally discuss the predictions of these results for the form factors in semileptonic heavy meson decays.

4.1. Simplest example: decay constant

The following analysis of the two–point function is particularly important as it determines besides the heavy meson decay constant, the parameter $\bar{\Lambda}$ as well. The analogue of the decay constant f_M in HQET is defined by

$$\langle 0|\,\bar{q}\,\Gamma\,h_v\,|M(v)\rangle = \tfrac{i}{2} F(\mu)\,\mathrm{Tr}\{\Gamma\,\mathcal{M}(v)\}\,. \tag{14}$$

In leading order $f_M\sqrt{m_M} \simeq F(\mu)$. The idea of QCD sum rules is to calculate the current–current correlator in two different ways. In HQET one defines [17]

$$\Pi(\omega) = i\int d^4x\, e^{ik\cdot x}\,\langle 0|\,\mathcal{T}\{J_M^\dagger(x), J_M(0)\}\,|0\rangle\ . \tag{15}$$

Here J_M is an interpolating current for the ground state heavy mesons $J_M = \bar{h}_v\,\Gamma_M\,q$ ($\Gamma_P = -\gamma_5$ and $\Gamma_V = \gamma_\mu - v_\mu$ for the pseudoscalar and vector mesons respectively), $k = P - m_Q\,v$ is the "residual" off–shell momentum [7], and $\omega = 2v\cdot k$. It is convenient to factor out the Lorentz structure of the two–point function, by defining $\pi(\omega)$ through $\Pi(\omega) = -\frac{1}{2}\,\mathrm{Tr}\{\bar{\Gamma}_M\,P_+\,\Gamma_M\}\,\pi(\omega)$.

Theoretical side: One side of the sum rule is a diagrammatic calculation of the correlator in terms of quark and gluon fields. As we approach resonances from short distances, nonperturbative effects induce power corrections that violate asymptotic freedom. These effects are taken into account in the Wilson operator product expansion (OPE)

$$\pi^{\text{theo}}(\omega) = \sum_n C_n O_n = \pi_{\text{pert}}(\omega) + \pi_{\text{cond}}(\omega) \tag{16}$$

through the so–called "condensates". These are vacuum expectation values (VEV-s) of gauge invariant local quark–gluon operators [14]:

$$\langle 0|\, \bar{q} q\, |0\rangle\,, \qquad \langle 0|\, \alpha_s G_{\mu\nu} G^{\mu\nu}\, |0\rangle\,, \qquad \langle 0|\, g_s \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q\, |0\rangle\,, \qquad \ldots \tag{17}$$

The perturbative contributions are depicted in the first line of Fig. 3, while the condensates are shown in the second and third lines (the gluon condensate $\langle \alpha_s GG \rangle$ does not contribute).

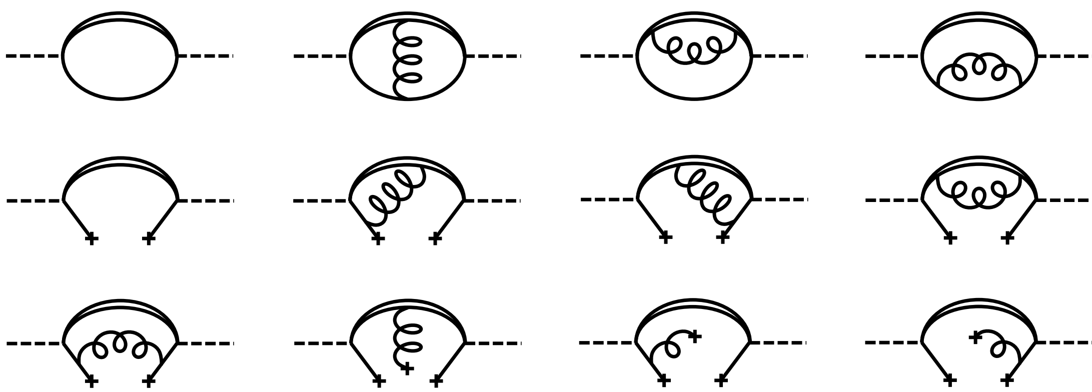

Figure 3: Feynman diagrams contributing to the sum rule for meson decay constants in HQET. The disjoint lines symbolize the nonvanishing VEV-s of the condensates (from [13]).

The condensates vanish by definition in the standard perturbation theory. Assuming that they can acquire non–zero VEV-s is a simple (but maybe very reasonable) modeling of the QCD vacuum. Since the condensates have different dimensions, Eq. (16) forms an expansion in inverse powers of ω. The coefficients C_n include, by construction, only short distance effects, the large distance contributions are accounted for by the vacuum to vacuum matrix elements. Strictly speaking, this gives rise to an uncertainty of the method, as we shall assume that C_n can be calculated perturbatively, while the nonperturbative physics is hidden solely in the VEV-s of O_n. These parameters are determined from various processes [14,15],

$$\begin{aligned} \langle \bar{q} q \rangle &= -\,0.013\,\text{GeV}^3\,, \\ \langle \alpha_s GG \rangle &= 0.04\,\text{GeV}^4\,, \\ \left\langle \bar{q}\, g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \right\rangle &= m_0^2\, \langle \bar{q} q \rangle\,, \quad m_0^2 = 0.8\,\text{GeV}^2\,, \end{aligned} \tag{18}$$

and then their values can be used as inputs in other calculations. Out of these, the gluon

condensate is the most poorly determined, however, it plays a negligible role in the sum rules we shall discuss. This value of $\alpha_s \simeq 0.3$ corresponds to the scale $\mu = 2\bar{\Lambda} \simeq 1\,\mathrm{GeV}$, which is appropriate for evaluating radiative corrections in the effective theory. Hence we write the correlator as $\pi^{\rm theo} = \pi_{\rm pert} + \pi_{\rm cond}$, and $\pi_{\rm pert}$ can further be rewritten as a dispersion integral

$$\pi_{\rm pert}(\omega) = \int d\nu \, \frac{\rho_{\rm pert}(\nu)}{\nu - \omega - i\epsilon} + \text{subtractions}\,. \tag{19}$$

Phenomenological side: The second calculation of the correlator is in terms of hadrons. $\Pi(\omega)$ is analytic in ω with discontinuities along the positive real axis. The correlator can be expressed as a sum over a complete set of physical intermediate states:

$$\Pi^{\rm phen}(\omega) = \sum_X \frac{|\langle X(v)|\, J_M \,|0\rangle|^2}{\omega_X - \omega - i\epsilon} + \text{subtractions}\,, \tag{20}$$

where the sum is over both discrete and continuum states, and $\omega_X = 2\,(m_X - m_Q)$. To evaluate the right hand side without a detailed knowledge of the spectrum and the matrix elements of the excited states, we employ local (in ω) quark–hadron duality. We separate the pole corresponding to the ground state meson, and approximate the contribution of higher resonances by an integral over the perturbative spectral density above the so–called continuum threshold ω_0. We find

$$\pi^{\rm phen}(\omega) = \frac{F^2(\mu)}{2\bar{\Lambda} - \omega - i\epsilon} + \int_{\omega_0}^{\infty} d\nu \, \frac{\rho_{\rm pert}(\nu)}{\nu - \omega - i\epsilon} + \text{subtractions}\,. \tag{21}$$

The QCD sum rules are obtained by matching the theoretical expression with the phenomenological one:

$$\frac{F^2(\mu)}{2\bar{\Lambda} - \omega - i\epsilon} = \int_0^{\omega_0} d\nu \, \frac{\rho_{\rm pert}(\nu)}{\nu - \omega - i\epsilon} + \text{subtractions} + \pi_{\rm cond}(\omega)\,. \tag{22}$$

In order to extract information about the ground state, one has to go to small values of $(-\omega)$ to enhance the relative weight of the low energy contributions. On the other hand, the theoretical calculation is only reliable when $(-\omega)$ is large. To achieve a balance between these contradicting requirements, an alternative way to probe $\pi(\omega)$ for small values of $(-\omega)$ is by taking derivatives. In the limit $-\omega \to \infty$ and the number of derivatives $n \to \infty$, one is sensitive to the behavior of the correlator at scales $T = -\omega/n$. This defines the Borel transformation

$$\frac{1}{T}\,\hat{B}_T^{(\omega)} = \lim_{\substack{-\omega, n \to \infty \\ -\omega/n = T}} \frac{\omega^n}{\Gamma(n)} \left(-\frac{d}{d\omega}\right)^n . \tag{23}$$

Employing this operator alters the sum rule (22) in such a way that

– possible subtraction terms are eliminated;
– contributions of higher dimensional condensates are factorially suppressed;
– higher resonance contributions are exponentially damped in the dispersion integral (the weight function $[\nu-\omega]^{-1}$ is replaced by $\exp[-\nu/T]$).

This yields the final form of the QCD sum rule:

$$F^2(\mu)\, e^{-2\bar{\Lambda}/T} = \int\limits_0^{\omega_0} d\nu\, \rho_{\mathrm{pert}}(\nu)\, e^{-\nu/T} + \hat{B}_T^{(\omega)}\, \pi_{\mathrm{cond}}(\omega). \tag{24}$$

To evaluate this sum rule, the continuum threshold ω_0, the Borel parameter T, and $\bar{\Lambda}$ have to be determined. Notice that by taking the logarithmic derivative of Eq. (24) with respect to T^{-1} we get a sum rule for $\bar{\Lambda}$. Next, we have to find the continuum threshold ω_0 such that $\bar{\Lambda}$ is stable with respect to variations of the Borel parameter T. Then we use this value of $\bar{\Lambda}$ to extract $F^2(\mu)$ from the sum rule (24). The range of ω_0 and T in which the predictions of the sum rule (for $\bar{\Lambda}$ and $F^2(\mu)$ in the present case) are independent of the value of T is called the "sum rule window". This provides a consistency check on the assumption of local duality, in the sense that if duality did not hold there were no reason why such a stability region existed (it can only result from cancelation of contributions on the theoretical against the phenomenological side of the sum rule). For the consistency of the calculation it is necessary that both sum rules be stable in the same range of ω_0 and T. Whether this happens is far from trivial and checks, in fact, the consistency of the approximations and the applicability of the method. Another assumption of QCD sum rules is that there exists a transition domain where the perturbative calculation is still reliable [$\alpha_s(T)$ is not too large] and the spectral density is sufficiently sensitive to the ground state. This can be assured by requiring that in the sum rule window the nonperturbative contributions to the sum rule be less than 30% of the perturbative ones, and that the pole contribution accounts for at least 30% of the perturbative part of the correlator.

Several authors calculated the decay constants of heavy mesons [17,18]. The symmetry breaking corrections (both from QCD and $1/m_Q$ effects) are large, resulting in significant deviations from the $f_B/f_D = \sqrt{M_D/M_B}$ scaling of the symmetry limit. We quote [17]

$$f_D = 170 \pm 30\,\mathrm{MeV}\,, \qquad\qquad f_B = 190 \pm 50\,\mathrm{MeV}\,, \tag{25}$$

which compares well with lattice results [27]. The mass parameter $\bar{\Lambda}$ that sets the characteristic scale of the $1/m_Q$ expansion is significantly higher than Λ_{QCD}: $\bar{\Lambda} = 0.57 \pm 0.07\,\mathrm{GeV}$ [17,13].

4.2. Universal functions

For the calculation of the universal form factors, we need to consider three–point correlation functions. The calculation, in principle, is very similar to the one outlined above. There are certain technical complications related to the applicability of local duality in two variables; different choices of the duality region to model the higher resonance states; and the possibility of introducing non–local condensates. These have all been discussed in detail in the literature [17,19]. The important point is that the physical predictions are obtained by taking the ratio of the three–point and the two–point sum rules. This assures that the normalization conditions at zero recoil are exactly fulfilled, independent of the value of the Borel parameter T and the continuum threshold ω_0. These sum rules are probably more accurate than the one for the decay constant, since some of the systematic errors drop out from such a ratio, and there is no dependence on the less accurately determined $\bar{\Lambda}$ parameter. It is, in fact, very promising that the position of the sum rule window is almost identical in the calculations of the decay constant, Isgur–Wise function, and the four subleading universal functions at order $1/m_Q$ ($\omega_0 = 2.0 \pm 0.3\,\mathrm{GeV}$, $T = 0.8 \pm 0.2\,\mathrm{GeV}$). Since the order α_s terms are large in the two–point function, to obtain a reliable result, it is imperative to include the order α_s corrections into the analysis of the three–point functions as well. The evaluation of the arising two loop diagrams require somewhat sophisticated calculational techniques [23,20]. By now, all parameters that appear in the heavy quark expansion of meson form factors have been calculated to order α_s/m_Q in the framework of QCD sum rules.*

<u>Isgur–Wise function:</u> Several authors calculated the Isgur–Wise function [17,19–21] (see: Fig. 4). Of special interest is its slope at zero recoil, described by the ρ^2 parameter. It turns out, however, that the sum rule prediction for this quantity is sensitive to the choice of the duality region, in particular to whether the continuum threshold ω_0 is allowed to depend on y [38]. Combining these uncertainties we conclude that QCD sum rules have a limited power in determining ρ^2, only constraining it to $0.6 < \rho^2 \lesssim 1.2$ [17,19]. This upper limit can only be realized by allowing ω_0 to depend on y, and it slightly exceeds the Voloshin bound $\rho^2 \lesssim 1$. In fact, the experimental data seem to prefer $\rho^2 \approx 1$ [37].

<u>$1/m_Q$ corrections:</u> For the four universal functions that appear at order $1/m_Q$ of the heavy quark expansion, the perturbative order α_s corrections are even more significant compared to the leading order calculation [22], than in the case of the two–point function sum rule [23,24]. These corrections can change certain results by as much as a factor of two. The form factors χ_2 and χ_3, which parameterize the matrix elements of the O_{mag} chromomagnetic operator in the effective Lagrangian, come out to be numerically negligible [23]. However, ξ_3 is of order unity; it is proportional to the Isgur–Wise function $\xi(y)$ to a very good approximation [24]. It

$\star$ This is not true for $\chi_1(y)$. However, this function does not affect any of the model–independent predictions of HQET.

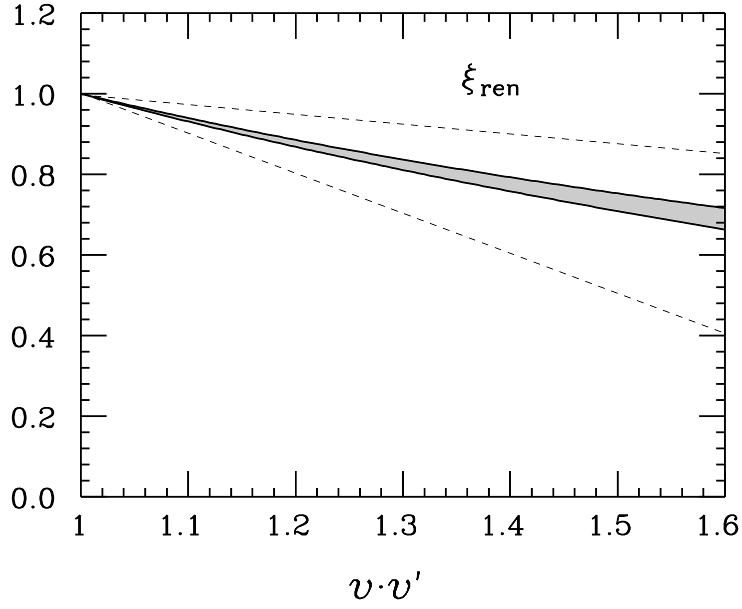

Figure 4: Prediction for the Isgur–Wise function. The dashed lines indicate the Bjorken and Voloshin bounds on the slope at $y = 1$. The shaded region shows only the uncertainty related to variations of the sum rule parameters (from [13]).

is convenient to present the result in terms of the renormalization group invariant ratio

$$\eta(y) \equiv \frac{\xi_3(y,\mu)}{\xi(y,\mu)} = 0.6 \pm 0.2\,, \tag{26}$$

which already includes a conservative estimate of the systematic uncertainties as well. This is the result that we have already discussed in Section 2, which makes the extraction of $|V_{cb}|$ surprisingly accurate from the kinematically suppressed $B \to D\,\ell\,\bar\nu$ decay mode.

In summary: QCD sum rules provide at present the theoretically most consistent framework to calculate the universal functions appearing in HQET, because:

- The Ward identities of the effective theory, *i.e.* the zero recoil conditions are *exactly* fulfilled, independent of the sum rule parameters;
- Due to the simple Feynman rules of HQET, two loop corrections can be included;
- It allows for systematic renormalization group improvement;
- There is a self–consistency check within the model on the assumption made.

Some of the uncertainties can be estimated in a reliable manner. These are the ones related to (a) the truncation of the power series in the OPE (condensates); (b) higher orders in the perturbative expansion; (c) numerical values of the condensates. It is much harder to estimate the systematic uncertainties inherent in the method, related to the assumption of local duality. Therefore, it is crucial to determine carefully the sum rule window and check the stability, which (if found) makes us believe that duality indeed holds. Due to these systematic uncertainties, the accuracy of the QCD sum rule method cannot be arbitrarily improved (unlike the case of

lattice calculations). Experience tells us that if all the above requirements are met, then the physical predictions of QCD sum rules are accurate to within 20–30%.

4.3. Form factors

The predictive power of the above results become more transparent if we translate them into predictions for the form factors measurable in $B \to D^{(*)}\ell\,\bar\nu$ decays. Throughout the following discussion we use the form factor basis defined in Ref. [39]. In this notation, in the vanishing lepton mass limit, $F_1(q^2)$ is the single measurable form factor in $B \to D\,\ell\,\bar\nu$ decays, while $A_1(q^2)$, $A_2(q^2)$, and $V(q^2)$ can be measured in $B \to D^{(*)}\ell\,\bar\nu$. In the infinite heavy quark mass limit [12]

$$\left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right]^{-1} A_1(q^2) = A_2(q^2) = V(q^2) = F_1(q^2)\,. \tag{27}$$

The finite masses of the b and c quarks introduce symmetry breaking effects. The predictions for these form factors are plotted in Ref. [13]. It is useful to define ratios of the three form factors measurable in $B \to D^*\ell\,\bar\nu$ decays:

$$R_1(q^2) = \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{V(q^2)}{A_1(q^2)}\,, \qquad R_2(q^2) = \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{A_2(q^2)}{A_1(q^2)}\,. \tag{28}$$

Clearly, in the heavy quark symmetry limit $R_1 \equiv R_2 \equiv 1$. In the following table we present the predictions of our QCD sum rule analysis in HQET [23,24,13], together with the quark models of ISGW [40], BSW [39], KS [41], and QCD sum rules in "full" QCD [42] (not including radiative corrections):

$B \to D^*\ell\,\bar\nu$	HQET + QCD SR	ISGW	BSW	KS	QCD SR
$R_1(q^2_{\rm max})$	1.35	1.01	0.91	1.09	1.31
$R_1(0)$	1.27	1.27	1.09	1.00	1.23
$R_2(q^2_{\rm max})$	0.79	0.91	0.85	1.09	0.95
$R_2(0)$	0.85	1.14	1.06	1.00	1.05

Table 2: Predictions for the form factor ratios at the endpoins of the spectrum.

The large values of R_1 is an almost model–independent prediction of HQET, in the sense that both the QCD and the $1/m_c$ corrections are positive, and the $1/m_b$ terms could only cancel the previous two, if the QCD sum rule prediction for ξ_3 failed with an order of magnitude. On the other hand, R_2 depends more strongly on ξ_3, and other models tend to give significantly higher values than ours. An experimental measurement of this quantity could distinguish between the above predictions.

5. Summary and Open Problems

In conclusion, we have seen how heavy quark symmetry reduces hadronic uncertainties by providing relations among heavy hadron form factors, and determining the absolute normalization of some of them at the kinematic limit point. Deviations from the predictions in this symmetry limit can be taken into account in the framework of a low energy effective theory. To calculate the universal functions appearing in HQET, QCD sum rules provide the theoretically most consistent framework. Of particular interest is the extraction of $|V_{cb}|$: the theoretical uncertainty has been reduced to less than 5% in the decay $B \to D^* \ell \bar{\nu}$, and it should be hardly larger in $B \to D$ decays. Of course, the experimental measurement of $B \to D \ell \bar{\nu}$ near zero recoil is more difficult. However, it would be an independent measurement providing checks on both the theoretical and the experimental analysis. The QCD sum rule determination of the universal functions that appear at order $1/m_Q$ in the heavy quark expansion gives predictions for the symmetry breaking effects. A measurement of the $R_{1,2}$ form factor ratios in $B \to D^*$ decays would distinguish between these sum rule predictions and other models.

In those cases when the final state does not contain a single heavy quark, HQS does not yield as restrictive relations as for heavy to heavy transitions. In particular, there is no absolute normalization of form factors. Still, different processes can be related to each other. This is the idea behind extracting $|V_{ub}|$ from a comparison of $B \to X \ell \bar{\nu}$ and $D \to X \ell \bar{\nu}$, where X is either π or ρ [43]. When $X = \pi$, additional constraints are provided by chiral symmetry and soft pion theorems. The main problems are (a) that the contribution of the B^* pole is large, and even dominant in the chiral limit; (b) the $1/m_Q$ terms are expected to be significant. If these difficulties are overcome then the determination of $|V_{ub}|$ from the experimental data may become much more reliable than that at present [44].

Here we only discussed exclusive semileptonic decays of heavy mesons. However, heavy quark symmetry also provides clues for accurate calculations of inclusive decays [36]. There are also exciting developments in progress to achieve a reliable, model–independent understanding of hadronic decays [45]. These theoretical developments will hopefully lead to a precise understanding of heavy hadron physics, ultimately allowing to distinguish the signatures of new physics in heavy quark systems (if such signatures exist) from the ever decreasing uncertainties in the predictions of the Standard Model.

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